Rationale and Syllabus Outcomes

Mathematics is a creative subject requiring abstract thought. Children naturally reason and use creative strategies when they seek patterns and relationships that will enable them to solve challenging unfamiliar problems. The generalisations they make can then be used to solve problems with the same mathematical structure.

Through the process of problem solving and class discussion of the strategies used, children will also develop skills they can use when faced with more unfamiliar problems, so by Stage 3 they will be able to:

- describe and represent mathematical situations in a variety of ways (MA3-1WM)
- select and apply appropriate problem-solving strategies in undertaking investigations (MA3-2WM)
- give valid reasons for supporting one possible solution over another (MA3-3WM)

This resource kit has been designed to assist in teaching problem solving strategies in preparation for the Maths Games. The sample problems are from Junior Maths Olympiad competitions in previous years. Further questions and solution methods can be found in the APSMO resource books available from www.apsmo.edu.au.

As with most competition problems, they can often be solved in many different ways. For this reason, different methods of solution will be suggested for each problem, with particular emphasis on:

1. Guess, Check and Refine

   This is a strategy that can help a student to get started. The student makes a reasonable guess of the answer, and then checks the guess against the conditions of the problem. If the first guess is not correct, at least one possible answer is eliminated and the student obtains further information that may lead to the correct answer. Beginners in particular are urged to use “Guess, Check and Refine” often, until they catch the “feel” of solving problems.

2. Drawing a Diagram

   If a problem is not illustrated, sometimes it is helpful for the student to draw a diagram. A picture may reveal information that may not be obvious just by reading the problem. It is also useful for keeping track of where the student is up to in a multi-step problem.

How to use these problems

At the start of the lesson, present the problem and ask the students to think about it. Encourage students to try to solve it in any way they like. When the students have had enough time to consider their solutions, ask them to describe or present their methods, taking particular note of different ways of arriving at the same solution.

Each question includes at least one solution method that the majority of students should be able to follow. By participating in lessons that demonstrate achievable problem solving techniques, students may gain increased confidence in their own ability to address unfamiliar problems.

Finally, the consideration of different solution methods is fundamental to the students’ development as effective and sophisticated problem solvers. Even when students have solved a problem to their own satisfaction, it is important to expose them to other methods and encourage them to judge whether or not the other methods are correct.
Maths Games – Example Problem 1.1

Each day Jeffrey earns $3 for washing the dishes. He can earn $5 instead by also sweeping the kitchen. After ten days, Jeffrey has earned a total of $36. On how many of these days did Jeffrey sweep the kitchen?

Similar Problems

1.1a) Admission to the local cinema is $3 for each child and $7 for each adult. A group of 12 people pay $64 admission. How many children are in this group?

1.1b) After returning from a holiday to the USA, Megan has some American coins: A 25c coins (quarters) and B 10c coins with a total value of $1.95, where A and B are both counting numbers. How many different values of A can Megan have?
Maths Games Example Problem 1.1 - Solution

Each day Jeffrey earns $3 for washing the dishes. He can earn $5 instead by also sweeping the kitchen. After ten days, Jeffrey has earned a total of $36. On how many of these days did Jeffrey sweep the kitchen?

Strategy 1: Guess, Check and Refine

The question is, on how many of these days did Jeffrey sweep the kitchen?

Firstly, Jeffrey worked for 10 days.

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Let’s guess that Jeffrey swept the kitchen on 6 days, for which he got $5 per day.

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How much is that? 6 \times 5 = $30.

On the other 4 days he only washed the dishes, for which he got $3 per day.

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How much is that? 4 \times 3 = $12.

So in total he would have earned $30 + $12 = $42.

But the question says he got $36, so that’s too much. Let’s change one of the $5 days to a $3 day.

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How much is that? 5 \times 5 = $25, and 5 \times 3 = $15.

So in total he would have earned $25 + $15 = $40. That’s $2 closer to the $36 target.

What happens if we change another $5 day to a $3 day? ($2 less again)

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And again? ($2 less again)

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How much is that? 3 \times 5 = $15, 7 \times 3 = $21, and $15 + $21 = $36. So it matches what it says in the question.

So: Jeffrey swept the kitchen on 3 days.

Strategy 2: Start with an extreme case.

Suppose Jeffrey swept the kitchen on none of the ten days. He would have earned $30.

This is $6 less than he actually earned.

Each day he sweeps the kitchen he earns an additional $2, so Jeffrey swept the kitchen on 3 days.

Let’s check: 3 \times 5 + 7 \times 3 = $36.
Maths Games – Example Problem 1.2

Toni picks a number and multiplies it by 3. She then adds 4 to the result and finally divides this new number by 2. Her final result is 14. With what number did she start?

Similar Problems

1.2a) Kristen has had her cat since it was a kitten. She said, “If you multiply my cat’s age by 4, and then divide by 12, you get 5. How old is my cat?”

1.2b) A starting number is multiplied by 4. Then 14 is added to the result. The new number is 6 times the starting number. What is the starting number?

1.2c) If 16 is added to one-third of a number, the result is three times the number. What is the number?
Maths Games Example Problem 1.2 - Solution

Toni picks a number and multiplies it by 3. She then adds 4 to the result and finally divides this new number by 2. Her final result is 14. With what number did she start?

Strategy 1: Guess, Check and Refine

The question is, with what number did she start?

Let’s guess that Toni picked 5 to start.

Firstly, she multiplies it by 3. \(5 \times 3 = 15\)

She adds 4 to the result. \(15 + 4 = 19\)

Finally she divides this new number by 2. \(19 \div 2 = 9\) remainder 1 (or \(9\frac{1}{2}\))

The final result was supposed to be 14. We got 9 remainder 1. What does this tell us?

- Maybe we should try a bigger number, because our result isn’t big enough.
- Maybe we should try an even number, to see if we can get a final result that doesn’t have a remainder.

2nd try: Let’s guess that Toni picked 8 to start.

Firstly, she multiplies it by 3. \(8 \times 3 = 24\)

She adds 4 to the result. \(24 + 4 = 28\)

Finally she divides this new number by 2. \(28 \div 2 = 14\)

That matches Toni’s final result. So Toni started with 8.

Strategy 2: Work backwards.

Toni ended on 14. We can work backwards to see what number she started with.

Let’s check: \(8 \times 3 = 24. 24 + 4 = 28. 28 \div 2 = 14\). So: Toni started with 8.